Key

Math 4

Name _____

Date_____

5-8 Product Rule

Complete the following:

At this point in your mathematical careers, you know what the derivative is, and you know how to find the derivative using the awesome difference quotient $\left(\lim_{x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}\right.$, in case you've forgotten$\left.\right)$ and a little quicker using the power rule. Problem is, the power rule only works for a limited number of functions. In this lesson we'll discuss a shortcut to finding derivatives for a different type of function.

Here are a couple examples of the new shortcut in action. Read them carefully and see if you can fill in the four parts of the formula after the examples. Keep in mind that when you find these derivatives in the future, you will need to simplify the answer.

Example 1: $f(x)=3x(4x^2)$

$f'(x)=3x\cdot 8x+4x^2\cdot 3$

Example 2: $f(x)=(\frac{1}{2}x^2-4x)(-5x+3)$

$f'(x)=(\frac{1}{2}x^2-4x)(-5)+(-5x+3)(x-4)$

Product Rule: If $f(x)=g(x)\cdot h(x)$ then $f'(x)=$ $g(x)\cdot h'(x)+h(x)\cdot g'(x)$

Now you try! $f(x)=(3x^2+5)(7x-4)$

$f'(x)=(3x^2+5)(7)+(7x-4)(6x)$

$=21x^2+35+42x^2-24x$

$=63x^2-24x+35$

In class practice: For each function, find the first derivative.

1. $f(x)=x^3(2x^2+3x-5)$

$f'(x)=x^3(4x+3)+3x^2(2x^2+3x-5)$

$=4x^4+3x^3+6x^4+9x^3-15x^2$

$=10x^4+12x^3-15x^2$

2. $j(x)=(2x+1)(x^2-3)$

$j'(x)=(2x+1)(2x)+(2)(x^2-3)$

$=4x^2+2x+2x^2-6$

$=6x^2+2x-6$

3. $q(x)=(\frac{1}{3}x^3+6x)(x^2-3x)$

$q'(x)=(\frac{1}{3}x^3+6x)(2x-3)+(x^2+6)(x^2-3x)$

$q'(x)=\frac{2}{3}x^4-x^3+12x^2-18x+x^4-3x^3+6x^2-18x$

$q'(x)=\frac{5}{3}x^4-4x^3+18x^2-36x$

4. $g(x)=(5x^2-3)^2=(5x^2-3)(5x^2-3)$

$g'(x)=(5x^2-3)(10x)+(10x)(5x^2-3)$

$=50x^3-30x+50x^3-30x$

$=100x^3-60x$

Homework, Day 1: Find the first derivative for each function.

SHOW ALL WORK ~~ON ANOTHER PIECE OF PAPER~~.

1. $f(x)=x^2(2x+5)$
2. $t(x)=(2x-3)(-4x^2)$
3. $y=(3x^2-2x+1)(2x-1)$
4. $f(x)=(3-4x)^2$
5. $g(x)=(2x^2+3x-2)^2$
6. $y=(x-1)(x^2+1)$

1.) $f'(x)=x^2(2)+2x(2x+5)$

$=2x^2+4x^2+10x$

$=\boxed{6x^2+10x}$

2.) $t'(x)=(2x-3)(-8x)+(2)(-4x^2)$

$=-16x^2+24x-8x^2$

$=\boxed{-24x^2+24x}$

3.) $\frac{dy}{dx}=(3x^2-2x+1)(2)+(6x-2)(2x-1)$

$=6x^2-4x+2+12x^2-6x-4x+2$

$=\boxed{18x^2-14x+4}$

4.) $f(x)=(3-4x)(3-4x)$

$f'(x)=(3-4x)(-4)+(-4)(3-4x)$

$=-12+16x-12+16x$

$=\boxed{-24+32x=32x-24}$

5.) $g'(x)=(2x^2+3x-2)(4x+3)+(4x+3)(2x^2+3x-2)$

$=2(8x^3+12x^2-8x+6x^2+9x-6)$

$=2(8x^3+18x^2+x-6)$

$=\boxed{16x^3+36x^2+2x-12}$

6.) $\frac{dy}{dx}=(x-1)(2x)+(1)(x^2+1)$

$=2x^2-2x+x^2+1$

$=\boxed{3x^2-2x+1}$